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Animal Cognition, Species Invariantism, and Mathematical Realism

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1 Introduction

What can we infer from numerical cognition about mathematical realism? In this chapter, I will consider one aspect of numerical cognition that has received little attention in the literature: the remarkable similarities of numerical cognitive capacities across many animal species. This Invariantism in Numerical Cognition (INC) indicates that mathematics and morality are disanalogous in an important respect: proto-moral beliefs differ substantially between animal species, whereas proto-mathematical beliefs (at least in the animals studied) seem to show more similarities. This makes moral beliefs more susceptible to a contingency challenge from evolution compared to mathematical beliefs, and indicates that mathematical beliefs might be less vulnerable to evolutionary debunking arguments. I will then examine to what extent INC can be used to flesh out a positive case for mathematical realism. Finally, I will review two forms of mathematical realism that are promising in the light of the evolutionary evidence about numerical cognition, *ante rem* structuralism and Millean empiricism.

2 The contingency challenge

Moral realism is the view that moral claims, such as "slavery is wrong" or "Jane is a good person," are about facts and that we know some of these facts. Moral facts are normative: they not only describe what is the case (e.g., slavery is wrong, or giving people their freedom is right), but also what ought to be the case (e.g., people should never be enslaved). Such facts are different from natural facts

(e.g., that water is composed of H_2O), but that does not make them any less true in the eyes of the moral realist. By contrast, moral antirealists contend that there are no moral facts. Traditionally, moral antirealists have argued that moral claims do not describe beliefs, but emotions (e.g., violence makes me feel bad; slavery makes me feel sorry for enslaved people). More recently, authors such as Sharon Street (2006, 2008) and Richard Joyce (2006) have argued against moral realism on evolutionary grounds. They worry that human moral intuitions and their resulting judgments are influenced by the peculiar evolutionary history of our species. Arriving at the correct moral beliefs, given the contingency of human evolution, would be a formidable and inexplicable instance of luck. It would be as if one set sail in the hope that the winds and tides will get one to Bermuda (Street, 2006: 121). As Street writes,

> There is a striking coincidence between the normative judgments we human beings think are true, and the normative judgments that evolutionary forces pushed us in the direction of making. I claim that the realist about normativity owes us an explanation of this striking fact, but has none. (Street, 2008: 207)

Street (2006) lists some moral concerns that are similar across many species, such as that survival is good, or that an obligation to care for one's offspring is greater than the obligation to help complete strangers. However, many other moral concerns are the result of the peculiar quirks of human evolution. For example, humans believe that helping unrelated strangers is a good thing, or that one should punish group members who do not follow social norms (Henrich et al., 2006). Such (proto)moral sentiments are not present in other primates (see e.g., Silk and House, 2011).

In the *Descent of Man* (1871) Darwin investigated, among many other topics, the evolution of the moral sense in humans. The overall project of that book was to show that although the difference between human cognitive capacities and those of others was substantial, including the human sense of morality, beauty, and religion, it was only a difference in degree and not in kind. Darwin sought to establish precursors of the moral sense in other animals. He conjectured that the evolution of moral capacities became unavoidable in cognitively complex social animals. As he wrote, "Any animal whatever, endowed with well-marked social instincts, the parental and filial affections being here included, would inevitably acquire a moral sense or conscience, as soon as its intellectual powers had become as well, or nearly as well developed, as in man" (Darwin, 1871: 71–72). But he also thought that their moral beliefs would vary depending on the social context in which members of the species would evolve. To illustrate

this point vividly, he offered the following thought experiment: if humans had evolved from animals with a eusocial structure, our moral beliefs would be very different from the ones we currently hold. For instance, "our unmarried females would, like the worker-bees, think it a sacred duty to kill their brothers, and mothers would strive to kill their fertile daughters; and no one would think of interfering" (Darwin, 1871: 73). This is because in eusocial societies the long-term survival of the group trumps concerns of individual workers. Eusociality has evolved three to eleven times independently in nature, in various clades including insects, shrimps, and mammals (West and Gardner, 2010). If there are rational creatures on other planets the kinds of actions we think are morally reprehensible could be obligatory for them and vice versa.

A moral realist could respond to this challenge by arguing that in eusocial structures, nests rather than individual workers are the right-bearers, so the difference may not be so vast after all. One could also argue that morality is a uniquely human domain, which does not even arise in other animals. Other animals have proto-morality at best: dispositions that lead them to helping behavior, or that lead them to prefer prosocial over antisocial individuals, but no explicit moral norms governing social interactions. Still, Darwin's bee thought experiment stresses the contingency of moral beliefs upon our specific evolutionary history. Our moral beliefs are not just the product of evolution; they are the peculiar outcome of human evolution, a haphazard process that has favored unique social structures and behaviors.

Lillehammer (2010: 365) terms challenges of this kind the *contingency challenge:* "We would have had very different beliefs if certain things about us had been different, even supposing the relevant ethical facts to remain the same." A related, but distinct, challenge is the *inflexibility challenge*, which says that we would have had the same beliefs, even if the relevant facts had been different. Street's (2006) Darwinian dilemma for moral realists stresses the inflexibility of our moral beliefs: even if pain were morally good in some realist sense, we would still be inclined to disvalue it because evolution through natural selection leads us to disvalue pain, as it decreases survival and reproductive success. Both kinds of challenge are part of a broader kind of purported failure, which spells bad news for moral realism, the *tracking failure* (Lillehammer, 2010). The evolutionary challenge to ethics does not amount to the claim that moral beliefs are likely not truth-tracking because they are the outcome of a long, evolutionary process (this is probably the case for all our beliefs, and thus would render the challenge trivial). Rather, it is the more specific claim that moral beliefs are not truth-tracking because they depend on contingent facts about our evolutionary history.

In this chapter, I will not be concerned with the evolutionary debunking literature on moral realism, but rather, with the question of whether evolutionary challenges to moral realism could be extended to the mathematical domain (see also, for example, Clarke-Doane, 2012, 2014; De Cruz, 2016). This fits in a broader literature of the so-called "companions in guilt" arguments where relevant features of moral realism are argued to occur in other domains (e.g., logic, perception, mathematics) (see, for example, Rowland, 2016). The basic outline of such an argument holds that these two domains fall or stand together: if a challenge to moral realism proves fatal, it will also be fatal for that other domain. The question here is whether evolutionary debunking arguments, if successful against moral realism, also damage mathematical realism. Mathematical realism—analogous to moral realism—is the claim that mathematical statements such as "2 + 2 = 4" are about facts. In order to assert this, the mathematical realist posits that mathematical entities (e.g., the natural numbers), relations, or structures exist. A dominant position in mathematical realism is platonism, which holds that mathematical objects exist. They are abstract entities that exist independently of human minds, cultural constructs, language, and symbols. Clarke-Doane (2014, manuscript) has connected the contingency challenge, normally applied to the question of moral realism, to the Benacerraf-Field challenge to mathematical realism. Benacerraf (1973) originally formulated the following objection to mathematical platonism: if mathematical objects are outside of space-time, how can we establish a causal link between these objects and the minds of mathematicians? How can the physical brain of the mathematician get access to these remote mathematical facts? Field (1989) reformulated the challenge in more general terms. Field's formulation does not require a causal link (which is a controversial requirement in any case), but hinges on the fact that mathematical realists cannot explain the reliability of mathematical beliefs:

> We start out by assuming the existence of mathematical entities that obey the standard mathematical theories; we grant also that there may be positive reasons for believing in those entities . . . Benacerraf 's challenge . . . is to . . . explain how our beliefs about these remote entities can so well reflect the facts about them . . . . If it appears in principle impossible to explain this, then that tends to undermine the belief in mathematical entities, despite whatever reason we might have for believing in them. (Field, 1989: 26)

How we should cash out "explain the reliability" is not easily resolved. Clarke-Doane (manuscript) argues that it should be spelled out in terms of safety: "In order to 'explain the reliability'" of our mathematical beliefs it is necessary to

show that we could not have easily had false ones (using the method that we actually used to form them), even if, had we, they would have been false." This formulation responds both to the contingency challenge: our mathematical beliefs are not easily false (for instance, because they are not contingent upon our evolutionary history in a way that is pernicious), and to the inflexibility challenge: our mathematical beliefs are not inflexible. They track something that is independent from them, and that could be false.

Clarke-Doane (2016) argued that mathematical realists might not face an access worry because mathematical beliefs are arguably safe. This might be because our "core" mathematical beliefs could be evolutionarily inevitable. However, in a more recent paper (manuscript) Clarke-Doane argues that it would be very hard for a mathematical realist to show that mathematical beliefs, say, about set theory, are safe, because it would be difficult to show that "we could not have easily believed different axioms of set theory." Indeed, the fact that mathematicians disagree about many core claims in every mathematical area shows that mathematical beliefs do not meet this criterion. For instance, Edward Nelson rejected the successor axiom (every natural number has a successor). Because mathematical beliefs are presumably less colored by irrelevant influences such as religion and cultural background, Clarke-Doane (2014) thinks that this puts mathematical realists in a worse position than moral realists, as the latter can at least explain away moral disagreements as a result of distorting irrelevant factors, but the former cannot.

This interpretation of the Benacerraf-Field challenge places the bar for realists quite high, some might argue, impossibly high. Moreover, some epistemologists have argued that safety is not a useful criterion for knowledge (e.g., Bogardus, 2014). Nevertheless, spelling out the Benacerraf-Field challenge in terms of safety can be useful if we consider the evolutionary origins of mathematical beliefs, in particular, numerical beliefs. In this chapter, I will not be concerned with set theory or other mathematical propositions, but with the evolutionary basis of our ability to form beliefs about numbers at all.

There is a large literature that supports the view that formal mathematics depends on evolved capacities to deal with number, which is collectively sometimes referred to as "the number sense" (e.g., Dehaene, 2011). Two capacities are hypothesized to underlie animals' ability to deal with numbers: the object file system (OFS) which might underlie our ability to enumerate and keep in working memory small collections of items (up to three or four) precisely, a capacity that is called *subitizing*, and the approximate number system (ANS),

which may underpin our capacity to estimate and compare larger collections (Feigenson et al., 2004).

Many cognitive scientists hold that the OFS and ANS lie at the basis of our ability to engage in more formal arithmetic abilities. Elizabeth Spelke (e.g., Spelke and Kinzler, 2007) has argued that ANS supplemented with language allows for the ability to engage in formal arithmetic. She finds support for the role of language in studies indicating that people who speak languages without exact number words cannot perform basic calculations exactly (e.g., 6 – 2 = 4), but their approximate numerical cognition is on par with numerate adults (Pica et al., 2004). Although it is limited in that it only allows for approximate numerical calculations, the ANS already allows for abstract numerical representations across modalities: preschool children can add and compare arrays of dots and sound sequences (Barth et al., 2005). Carey (2009) sees the OFS at the root of more formal arithmetical capacities. Her bootstrapping account emphasizes the role of subitizing in children's ability to learn the successor function in arithmetic. Children learn to associate the meanings of the first words in a count list (in English, "one," "two," and "three") with collections of one, two, and three items, which they can subitize. This explains why children tend to learn the meanings of number words in the same order: they first become one-knowers, then two-knowers, next three-knowers, and very occasionally, four-knowers. But because subitizing stops at three or four, they need to make an inductive generalization to learn the next words in the counting sequence. According to Carey, children then make the following induction: if "x" is followed by "y" in the counting sequence, adding an individual to a set with cardinal value x results in a set with cardinal value y.

The idea that these two capacities play a critical role in our ability to engage in formal arithmetic is not universally accepted (see for example, Rips et al., 2006, Rips et al., 2008). Some authors have argued that nonnumerical sensory properties, such as visual density and circumference, can explain the animal data and have questioned the existence of the ANS (e.g., Gebuis et al., 2016). That being said, the ANS and the OFS are still the predominant theories to explain animal numerical cognition. Authors such as Lourenco et al. (2012) have argued that people's ability to engage in approximate arithmetic (both symbolic and nonsymbolic) correlates with their mathematical abilities. This has been confirmed in a recent meta-analysis, although the correlation between mathematical skills and symbolic numerical abilities is stronger (Schneider et al., 2017). If formal arithmetic is dependent (in some causal or psychological sense) on the evolved number sense, it becomes relevant for mathematical realists to explore it in

more detail. In particular, explaining the reliability of our mathematical abilities will involve reference to the number sense and the way it forms beliefs about magnitudes.

3 Invariantism in numerical cognition

Numerical cognition is a well-researched domain of higher cognition. While obviously it is not identical across species (for one thing, humans use Arabic numerals whereas mosquito fish do not), I will here examine striking similarities between the numerical capacities of animals from a wide variety of species and clades, which I will call INC. I will here look at four features of numerical cognition across species to argue the case for INC: numerical cognition is present in many different animal species, including in animals with small, simple nervous systems such as insects and spiders (3.1), it plays a crucial role in animal adaptive decision making (3.2), it shows similarities in computational characteristics and limitations across species (3.3) and, to the extent that it has been investigated, there is evidence that numerical cognition is the result of convergent cognitive evolution rather than common descent (3.4).

3.1 Numerical competence is present in a wide variety of clades

Most research on numerical competence has been conducted with primates, including rhesus monkeys, capuchin monkeys, and chimpanzees. For example, rhesus monkeys are able to order collections of items from 1 to 9 (Brannon and Terrace, 1998). Other mammals, including brown bears and dogs, are also capable of discriminating numerosities. For example, brown bears were trained to select among two screens the display that had the largest number of dots (even if sometimes that meant the overall lowest surface area, because the dots were smaller), using food reinforcements (Vonk and Beran, 2012). Domestic dogs were tested using a violation-of-expectation paradigm, where they saw simple calculations including "1 + 1 = 2," "1 + 1 = 1," and "1 + 1 = 3." Dogs looked longer at the incorrect outcomes, which is interpreted as showing they did not expect the incorrect outcomes and thus know that 1 + 1 = 2 (West and Young, 2002). Birds, including pigeons, chickens (even newborn chicks), and crows, are capable of calculating and estimating collections of items (e.g., Scarf et al., 2011, Ditz and Nieder, 2016, Rugani et al., 2008). Although not all experiments control for nonnumerical cues, such as the total surface area or the density of displays, many

experiments have done so. For example, mosquito fish can discriminate between smaller (e.g., 3 vs. 2) and larger shoal (e.g., 8 vs. 5), even when controlling for the density of the fish and the overall space occupied by the group (Dadda et al., 2009). For larger groups (e.g., 8 vs. 4), total area and the amount of movement of the fish in both groups matter (Agrillo et al., 2008). Such experiments strongly suggest that it is numerical cues—rather than nonnumerical continuous variables—that animals are responsive to.

A recent domain of inquiry is numerical competence in insects and spiders (see Pahl et al., 2013 for a review). Although insects, such as bumblebees, have small nervous systems, they are very adept at integrating complex information, such as the relative returns of nectar by particular types of flowers, even depending on times of the day and the probability of yields (Real, 1991). Numerical information is one such source of information that insects and spiders use in their everyday ecological decisions. *Portia africana* spiders, for example, practice communal predation, sharing their prey with another resident conspecific. Juvenile *Portia africana* prefer to settle when there is one conspecific present, preferring this outcome to zero, two or three conspecifics (Nelson and Jackson, 2012). Dacke and Srinivasan (2008) designed a carefully controlled experiment where bees were trained to fly in a long tunnel where five landmarks, consisting of identical yellow strips, were placed at irregular intervals. A feeder was hidden at one of those landmarks. In the test condition, the researchers examined whether bees would look for the feeder close to the landmark where they were previously trained to find the feeder in. The bees' accuracy was very high up until landmark 3, but became more erratic at 4 and 5. Bees not only are able to discriminate numbers sequentially, but also to visually discriminate different numerosities of displays. They can match displays of two blue dots and two yellow stars, and can do so up to 3, and their performance drops at chance level at 4 (Gross et al., 2009). This is a striking similarity to other animals, suggesting bees may be subject to the same limitations of the OFS as human infants (Starkey and Cooper, 1980).

3.2 Numerical cognition plays a crucial role in animal adaptive decision making

While early authors writing on numerical competence in animals tended to dismiss it as a last resort, to which animals only turn if there is no other information available (Davis and Pérusse, 1988), the current consensus is that animals use their number sense in adaptive decision making. The best-studied ecological situation in which animals rely on numerosities is food choice:

given that they need to travel to a source of food and use up time and energy doing so, it makes sense to go to the source that has the most food. Research indicates that animals tend to "go for more," selecting maze-arms, feeders, and other experimental setups that have the largest number of food items. For example, free-ranging adult salamanders placed in a T-shaped enclosure that could choose between the ends containing either one or two live flies, or two or three live flies, chose the arm of the enclosure with the most flies (like other amphibians, salamanders can only visually see small stimuli if they move). However, they showed no preference if the choice was between 3 and 4 or 5 and 6, again revealing limits to the OFS (Uller et al., 2003).

Petroica australis, a food-caching songbird, shows sophisticated reliance on numerical information when storing, retrieving, and pilfering caches of food (mealworms). The birds could watch food being put in a pair of artificial cache sites, and could choose one of them. They were successful in finding the cache with the most mealworms (experimenters controlled for duration and other nonnumerical confounds) in caches up to twelve items. The experimenters also did a violation-of-expectation experiment, where birds watched a number of mealworms being stored, but only a subset was findable, and they examined whether these birds would take a longer time searching for the remaining worm(s). This study revealed that birds looked longer in 2 versus 1 in 3 versus 2, but not in 8 versus 4 conditions, perhaps because they were subject to the limitations of OFS which is especially operative for keeping numerosities in working memory (Hunt et al., 2008).

Animals also use numerical information for selecting their territory (Nelson and Jackson, 2012), and for choosing whether or not to attack a rivaling group, based on a comparison of that group's size and the own group's size (e.g., McComb et al., 1994 for a study with wild lionesses). Shoaling fish choose to aggregate with shoals based on their perceived size, for example, guppies prefer a shoal of 8 over a shoal of 4 to aggregate with (Bisazza et al., 2010).

3.3 Similarities in computational characteristics and limitations

As we have seen, the OFS and the ANS are the dominant ways to explain human numerical cognition. Both have specific limitations and characteristics. The OFS is accurate for collections of items up to 3 or 4. It allows animals to make comparisons and calculations across modalities. For example, rhesus monkeys (Jordan et al., 2005) and human infants (Jordan and Brannon, 2006) can match the number of voices they hear to the correct number of speaking heads they

see on a monitor. It also supports addition and subtraction. Infants, as well as domestic dogs, show surprise at unexpected additions and subtractions, such as $1 + 1 = 1$ or $2 - 1 = 2$ (Wynn, 1992, West and Young, 2002). Above 3 or 4, the OFS is not able to make calculations or comparisons anymore. For example, chicks can discriminate between displays of 1 and 2, and between 2 and 3 items, but not between 3 and 4, or 4 versus 5, or 4 versus 6 (Rugani et al., 2008), although chicks can tell the difference between larger numbers when the ratio difference between them is large enough, for example, 2 versus 8 and 8 versus 32 (Rugani et al., 2015). We saw above that this limitation was also observed in bees, salamanders, and human infants. Human numerate adults can, of course, distinguish between collections of 3 and 4, or 4 and 6. Yet, even adults are subject to the limitations of the OFS: they are much more accurate in enumerating small collections of items (up to 3) than larger collections, with a steep decline in precision after 3 (Revkin et al., 2008). The explanation for this limitation of the OFS is that there are inherent limitations to working memory. The OFS works by putting mental representations of discrete objects (e.g., two bananas, one sound and one dot) in a placeholder format as slots that are kept in working memory (Feigenson and Carey, 2005).

The ANS, unlike the OFS, does not have a strict limit on how much it can represent, although experimental setups typically stay under 100. This system handles the approximate representation of numbers, and, like the OFS, it can work across modalities, and it supports addition and subtraction (Barth et al., 2005). Next to these features, its outputs show the Weber-Fechner signature: the discriminability of two magnitudes (numerosities) is determined by their ratio. As a result, numerical judgment improves with increasing distance (e.g., it is easier to discriminate 2 from 8 than 7 from 8, not only if this is presented as collections of dots but even in symbolic format, see Moyer and Landauer, 1967 for the first classic study to show the distance effect in symbolic format).

Comparative research indicates that rhesus monkeys' performance on approximate arithmetical tasks is similar to that of college students. Students and rhesus monkeys were required to mentally add a number of dots and select a display that showed the sum (e.g., for displays of 1 and 7 dots, the display containing 8 dots had to be selected). Next to the display showing the correct sum (e.g., $1 + 7 = 8$) there was a distractor display that contained an incorrect number of dots (e.g., $1 + 7 = 5$), which had a cumulative surface area close to the correct solution. Although adults were more correct (94 percent correct answers, compared to only 74 percent for the monkeys), their response patterns were very similar, showing similar sensitivity to the ratio between the numerical values of

the sum and choice stimuli, in line with the Weber-Fechner law (Cantlon and Brannon, 2007). In a direct comparative study, pigeons performed on a par with primates in numerical tasks such as ordering cards with different numbers of items in ascending order, showing very similar distance effects, that is, better performance if numerosities lie further apart (Scarf et al., 2011).

There have been a few systematic studies that have examined whether the ANS in nonhuman animals other than primates obey the Weber-Fechner law. Gómez-Laplaza and Gerlai (2011) showed that angelfish (*Pterophyllum scalare*) can choose the larger of two shoals, and that their number discrimination is sensitive to the ratio difference between the two groups: for example, they prefer to aggregate with the larger shoal if the differences are 4:1 (e.g., 12 vs. 3), 3:1 (9 vs. 3), and 2:1 (8 vs. 4), but not at smaller ratio differences, for example, 1.5:1 (9 vs. 6 and 6 vs. 4). Carrion crows (*Corvus corone*) can discriminate numbers up to 30 (in displays that controlled for total surface area) in line with the Weber-Fechner law (Ditz and Nieder, 2016). While more research would need to be carried out to see how far this generalizes, the research so far supports similar cognitive mechanisms of OFS and ANS underlying animal numerical competence in a wide range of species.

3.4 Neural correlates of numerical cognition show evidence of convergent evolution

In human brains, several areas of the neocortex are associated with numerical cognition, in particular the bilateral intraparietal sulci. This area is active when adults engage in calculation with Arabic digits and dots (e.g., Dehaene et al., 1999) or even when participants are merely passively looking at or listening to Arabic digits or number words (Eger et al., 2003). The intraparietal sulci are also active in four-year-olds and in adults when presented with visual displays of collections of items that differ in number (Cantlon et al., 2006). Homologous areas in the primate parietal cortex and prefrontal cortex are responsive to numerosity. Recordings of single neuron responses in the brains of monkeys show that there are number-sensitive neurons in the lateral prefrontal cortex and the intraparietal sulcus of the posterior parietal cortex. These number-sensitive neurons selectively respond to a specific number of items in visual dot displays, including zero. Their response does not vary with other spatial features, such as the size of dots, but seems to be number-specific. While they preferentially fire at a given number of dots (say, 3), they will also respond, albeit less frequently, to other numerosities (say, 2 or 4), with response patterns following a Gaussian curve around the preferred numerosity (Tudusciuc and Nieder, 2007).

Bird numerical cognition is situated in the endbrain, more specifically in the nidopallium caudolaterale. The neurons of crows in this part of the brain fired selectively for different numerosities, just like they did in rhesus monkeys, for example, a neuron selectively tuned to 4 items also responded, but to a lesser extent, to 3 and 5 items (Ditz and Nieder, 2015). Primates and birds have markedly different brain structures. Their last common ancestor lived about 300 million years ago, at a time when the six-layered neocortex (which hosts, among others, the neurons responsible for numerical cognition) had not evolved yet in mammals. Thus, the similarities between crows and rhesus monkeys in neural representation of number show a striking convergent evolution.

The similarities between insect and mammalian (including human) numerical cognition cannot be due to homologous neural structures either. The European honeybee only weighs 0.1 g, and its brain only weighs 0.001 g, with a total size of 1 cubic mm^3, and about 1 million neurons. Compared to the human brain with its 100 billion neurons, it has only 1/100,000th of the number of human neurons. The main functions relating to memory and adaptive decision making are situated in the mushroom bodies and the central complex, so this is likely also where numerical cognition takes place. Unfortunately, it is not possible at present to find the neural correlates for numerical cognition in such a small brain (see, however, Greco et al., 2012 for recent advances in scanning brains of live bees). Given these similarities in processing, in spite of very different neural implementation, insect numerical cognition presents another case of convergent evolution.

4   The metaphysical significance of INC

The behavioral and neural similarities in the numerical cognition of a wide diversity of species and clades are a remarkable phenomenon, which I termed INC. It cannot be explained by homology (similarities due to a shared ancestral trait) given how divergent insect, avian and mammalian brains are. If homology cannot explain INC, what alternative do we have? Homology is often contrasted to homoplasy (similarity due to independent evolution), but homoplasy is a portmanteau term for several distinct evolutionary patterns (Hall, 2013). One of these is convergent evolution, when similar features evolve independently in different species as a result of similar evolutionary pressures. For example, insects, birds, and bats developed wings that help them to escape predators or pursue prey. INC is a good candidate for convergent evolution: a trait that emerged in diverse clades as a result of similar evolutionary pressures. An

alternative explanation for INC is homology, when traits that evolve through convergent evolution share a similar genetic regulatory apparatus. Examples include the Pax6 gene, which helps regulate vision in mollusks, vertebrates, and insects, and the FoxP2 gene, which regulates human language development and song production in songbirds (Scharff and Petri, 2011). However, even in these cases of deep homology there is considerable independent evolution to accommodate anatomical differences (e.g., the eye structure of insects versus mammals). Moreover, if the structures in question were not adaptive, it is unlikely that these deep homologies would have occurred. For example, the Pax6 gene regulates the prenatal development of eyes, such as the iris, and its function can be explained by the fact that seeing is adaptive. Thus, even if a deep homology underlies numerical competence in these widely divergent clades, the similarities between them remain striking.

Some of the convergence in numerical cognition across clades likely has to do with constraints in computation and memory, including the Weber-Fechner signature and the limitations of the OFS. Nothing of mathematical interest happens when natural numbers become larger than 3, it is just a limitation of working memory. Why would animals be better at discriminating smaller numerosities, and why would the ratio difference be more relevant than the absolute difference? The difference between small numbers is often more ecologically significant than that between large numbers, for example, to a hungry foraging monkey, it is more relevant to see the difference between a patch with 1 fruit versus 2 fruits than that it is to be able to distinguish between 11 and 12 fruits. This ecological function of numerical cognition leads me to posit the following claim: INC presents substantial evidence for mathematical realism. It indicates that animals are tracking something in the environment (numerosities), and realism is the best explanation for numerosities.

In an earlier paper (De Cruz, 2016), I outlined an indispensability argument for mathematical realism from numerical cognition. I proposed that the best explanation for numerosities involves numbers—animals make representations of magnitude in the way they do because they are tracking structural (or other realist) properties of numbers. This fits in an ongoing discussion on whether physical phenomena have genuine mathematical explanations. Baker (2005, 2015) has argued that this is the case, citing such cases as the primeness of the life cycle of insects that are members of the genus *Magicicada* and structural properties of honeycombs. In the case of *Magicicada*, their life cycles are either 13 or 17 years. These consist of a long phase they spend as larvae underground and a brief adult phase spent above ground when they reproduce. The primeness of

their life cycles makes it less likely that the life cycles of predatory species would intersect with them, thus increasing their reproductive success. Primeness is a mathematical property that plays a relevant role in the biological explanation for why their life cycles have these durations. Such examples are used to bolster the case for platonism about mathematical objects.

I will not reiterate these arguments here, but instead will consider Clarke-Doane's (2014, manuscript) more recent challenge. Now, the mathematical beliefs Clarke-Doane targets are those of professional mathematicians, such as the axioms of set theory, rather than more elementary beliefs such as that 7 is prime, or that 2 + 2 = 4. Indeed, he is happy to concede that the latter would be safe, just like the belief that burning babies for fun is wrong is true for any moral non-error-theorist (Clarke-Doane, 2014). However, the evolutionary challenge against mathematical realism targets those more basic beliefs too, just like evolutionary debunking arguments against moral realism challenge fundamental moral beliefs such as that pain is bad.

With INC we have a clear disanalogy between mathematics and morality: the proto-moral beliefs of different species are divergent, whereas numerical cognition is invariant across species. This makes numerical cognition less susceptible to the contingency challenge that has been proposed against moral realism. In the case of moral realism, one can see how our beliefs would easily have been different if our evolutionary history had gone a different way. Our moral beliefs could have easily been false (assuming a nonnaturalist form of moral realism[1]), but our mathematical beliefs could not have been. This is because evolution has shaped our minds (as well as those of bees, crows, rhesus monkeys, chicks, angelfish, etc.) to track numerical information. Similarities in numerical cognition across a wide range of unrelated species require some explanation, and mathematical realism can provide this explanation straightforwardly, namely what animals are tracking are mathematical truths/structures. I am not arguing that antirealists cannot explain INC. Nevertheless, the antirealist would need to explain why unrelated animals such as salamanders, bees, crows, angelfish, and rhesus monkeys (and of course humans), would be able to track discrete quantities in their environment, would be able to do so across modalities, and would use this information to inform their adaptive choices. INC thus shifts the burden of proof in the direction of the antirealist.

One can, of course, resort to highly contrived scenarios where animals have adaptive responses, such as choosing the most numerous shoal or cache of mealworms, without tracking mathematical truths. Plantinga's (1993) evolutionary argument against naturalism famously argued that animals can

have the right adaptive behaviors without truth-tracking beliefs, for example, a hominin who runs away from a tiger (adaptive response), but does so because he believes the tiger is cute and he wants to pet it, but he also believes that the best way to pet it is to run away from it (maladaptive belief). While such scenarios are metaphysically possible (and some have outlined them for the case of numerical beliefs, for example, Clarke-Doane, 2012), they are not very plausible. An error-theorist would have to come up with a scenario for each case of evolved numerical cognition (which, to the best of our knowledge, has occurred independently at least in insects, birds, mammals, and fish) where somehow wrong or irrelevant mathematical beliefs would lead to the right adaptive responses. At present, there is no satisfying positive case for mathematical antirealism that accounts for INC without resorting to arcane scenarios.

5 Which form of realism does the animal cognition literature support?

The Benacerraf-Field challenge to mathematical realism asks realists to explain the reliability of mathematical beliefs. This is not as demanding as outlining a causal account (which would be impossible under some forms of realism in any case), but requires us to show that our evolved mathematical beliefs are safe from error. I have argued in previous work (De Cruz, 2016) that Shapiro's (1997) *ante rem* structuralism is a possible candidate in the light of evolution. One reason to look more closely into realist structuralist accounts is that authors in this field, such as Shapiro (1997), have made substantial efforts to explain how their account would work in a naturalistic framework. Moreover, *ante rem* structuralism provides a straightforward account of reference and semantics, and can provide an account of mathematical structure irrespective of the agent cognizing it, which makes the approach suitable for our explanation of numerical cognition across species. To summarize, *ante rem* structuralism holds that non-applied mathematics is concerned with structures that are conceived of as abstract entities (platonic universals), that is, structures that exist independently and prior to any instantiations of them. *Ante rem* structuralists do not specify the precise nature of these entities, but rather focus on the role they play. Numbers are positions in a certain structure, and can be discerned in the environment as patterns. The bootstrapping account (Carey, 2009) can offer a glimpse of how we can have mathematical beliefs that are safe, and that track mathematical structures. According to this account, young children learn

to recognize the 1, 2, and 3 pattern, thanks to their OFS, which allows for exact discrimination of numerosities up to 3. Since the OFS is very precise, learning the 1, 2, and 3 pattern is a reliable process (at least in neurotypical children who do not suffer from dyscalculia). The children learn the remaining natural numbers through a process of induction. This part of the learning process is stable thanks to the abundant cultural scaffolding (e.g., counting songs) and feedback (e.g., parents correcting their child, or helping their child to count a given collection of items) children receive (see also De Cruz, 2018). In this way, an *ante rem* structuralist can explain the reliability of our natural number concepts to track mathematical truths.

Another plausible realist (non-platonist, in this case) account that is compatible with INS, is Millean empiricism. Kitcher (1984) revived this position, arguing that mathematical epistemology should seek inspiration from how children learn arithmetic. It thus fits well in a naturalistic account of mathematical cognition. A closely related view is Aristotelian realism, recently defended by Franklin (2014). Mill (1843, 165) proposed that numbers are properties of physical aggregates:

> When we call a collection of objects two, three, or four, they are not two, three, or four in the abstract; they are two, three, or four things of some particular kind; pebbles, horses, inches, pounds' weight. What the name of number connotes is, the manner in which single objects of the given kind must be put together, in order to produce that particular aggregate.

In this view, numerical cognition detects high-level, general properties of aggregates, for example, an angelfish that chooses a shoal of 7 fish over 3 fish is detecting the high-level general property of aggregates that 7 > 3. According to Mill, we do not need to invoke the existence of 3 and 7, separate from their concrete instantiations in the physical world. Millean empiricism does not presuppose platonist ontology, but it is nevertheless a realist ontology (see Balaguer, 1998, chapter 5), because it regards the laws of arithmetic as highly as general laws of nature.

A common objection to Millean empiricism is that aggregates do not have determinate number properties. For example, a group of lions can be divided into many different parts, for instance, it is composed of 7 lions, 28 legs, etc. Kessler (1980) responds to this problem by arguing that in Mill's account, numbers are not properties of aggregates, but relations that hold between aggregates (e.g., the pride) and properties of those aggregates (e.g., individual lions). Infants

and animals are successful at finding the relevant properties of aggregates in numerical tasks, for instance, they can compare the number of speakers they see with the number of voices they hear (Jordan and Brannon, 2006). When they are presented with a collection of objects (e.g., an array of dots) infants seem to be less able to detect a decrease or increase in the individual object's size, than they are to detect a change in numerosity. They need as much as a fourfold change in size to notice it, as revealed by a longer looking time. This suggests that once infants attend to numerosity, they disregard the physical particulars of the items that constitute them (Cordes and Brannon, 2011). In line with Millean empiricism, they can make high-level generalizations about numerosities that go beyond the physical properties of aggregates. Given that the world at our scale mostly consists of separable objects, there may have been an evolutionary advantage of making high-level generalizations about numerosities, along the lines of separable objects as we and other animals encounter them in daily life (see also Dehaene, 2011: 231).

Ante rem structuralism and Millean empiricism are two realist ontologies that are compatible with the evolved features of numerical cognition. Both meet the Benacerraf-Field challenge of explaining the reliability of numerical representations. For the structuralist account, direct interaction with structures is not required to know numbers, and for Millean empiricism, numerosities form a high-level generalization of the properties of discrete middle-sized objects.

6 Conclusion

In this chapter, I have argued that mathematics and morality are disanalogous in an important respect. Mathematical beliefs seem to be less contingent upon our peculiar evolutionary history than moral beliefs are. I have presented evidence for invariantism in numerical cognition: numerical cognition occurs across many animal clades, including insects, fish, amphibians, birds, and mammals, and is, according to the dominant theories on numerical cognition, subserved by two systems: the ANS, which deals with larger collections of items through approximation, and the OFS, an exact system for small numerosities up to 3 or 4. Numerical information plays a crucial role in animal decision making. Animals across widely different clades show similar cognitive limitations and strategies in dealing with numbers, including an ability to deal with numbers across modalities. Neural evidence suggests multiple instances of convergent

evolution. If animal minds have hit upon these solutions so many times independently, this would be a formidable coincidence which antirealists would need to explain. Of course, INC also requires an explanation under the assumption of mathematical realism. In particular, the Benacerraf-Field challenge asks the mathematical realist to explain the reliability of mathematical beliefs. If this were in principle impossible to achieve, this would undermine our mathematical beliefs, according to Field (1989). The Benacerraf-Field challenge can be cashed out in terms of safety: the realist needs to show that we could not easily have had false mathematical beliefs.

I showed that ante rem structuralism and Millean empiricism provide a solution to the Field-Benacerraf challenge: they can explain the reliability of animal numerical beliefs, and thus by extension of human mathematical beliefs that are based upon them, such as the belief that 11 follows 10, or that 7 is prime. My argument does not provide an evolutionary justification of more formal mathematical beliefs, such as those involved in set theory.

Acknowledgments

Note

1   Some naturalistic forms of moral realism are less susceptible to the contingency challenge, in particular the neo-Aristotelian approach to morality (as e.g., outlined by Foot, 2001). According to neo-Aristotelians, what counts as a good human life and human flourishing is the truth-maker of moral claims. Humans have, as evolved creatures, certain limitations on the conditions that will make them thrive and flourish. The role of the ethicist is to find out how to fulfill these conditions. Fitzpatrick (2000) has challenged the neo-Aristotelian account by pointing out that not all evolved features lead to flourishing; for example, male elephant seals fight to gain control of large harems, which makes evolutionary sense but does not seem to contribute to their well-being. However, as Lott (2012) has countered, the neo-Aristotelian approach does not look at animals from the outside, but instead from the inside of life forms. In that respect, it would seem that it is "good" for bee queens to kill their fertile daughters, to harken back to Darwin's example.

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