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TOWARDS A DARWINIAN APPROACH TO MATHEMATICS

ABSTRACT. In the past decades, recent paradigm shifts in ethology, psychology, and the social sciences have given rise to various new disciplines like cognitive ethology and evolutionary psychology. These disciplines use concepts and theories of evolutionary biology to understand and explain the design, function and origin of the brain. I shall argue that there are several good reasons why this approach could also apply to human mathematical abilities. I will review evidence from various disciplines (cognitive ethology, cognitive psychology, cognitive archaeology and neuropsychology) that suggests that the human capacity for mathematics is a category-specific domain of knowledge, hard-wired in the brain, which can be explained as the result of natural selection.

KEY WORDS: philosophy of mathematics, cognitive science, animal cognition, cognitive archaeology, evolution

1. WHY A DARWINIAN APPROACH TO MATHEMATICS?

The philosophy of mathematics has not paid much attention to the biological context of mathematics, although it stands to reason that mathematicians are biological organisms, and that they produce mathematical theory and practice with their brains. Human mental faculties have been the object of scientific research for centuries. Recently cognitive scientists (e.g. Barkow et al., 1992; Cosmides and Tooby, 1994; Sperber, 1996) have placed human mental faculties into an evolutionary perspective. The ability to do mathematics has traditionally been considered as a human faculty without any survival value, and thus not fit for a Darwinian approach. For example, Wallace, with Darwin the co-founder of the theory of natural selection, was so impressed by the power and scope of the human mind, that he could not believe it could have arisen through natural selection

(Cartwright, 2000, 17). At first sight mathematical abilities do not seem vital to survive and reproduce. In this paper, I shall investigate the epistemology, explanatory power, and methodology of a Darwinian approach to mathematics.

1.1. *Setting the Metaphysical Framework: Top-down Versus Bottom-up Approaches*

A Darwinian approach to human mental faculties is a typical *bottom-up* type of explanation. To understand its full implication it is necessary to examine the metaphysical nature of this type of explanation. When studying complexity (e.g. in physical or biological processes) a scientist may apply either one of the following metaphysical frameworks, top-down or bottom-up. A top-downer believes that every form of complexity arises from a higher faculty, usually God or some Platonic world. The traditional Neo-Platonist Christian worldview is typically top-down. It conceives the world as a hierarchical system where every higher step influences a lower one, but not vice versa (Wildiers, 1989). God is on the top of this ladder, and influences every lower step. The angels are below God (they cannot influence him, while he has control over them), and the celestial objects are below the angels (which is why they were supposed to be controlled by the angels, and were themselves thought to influence men). Philosopher of science Radcliffe (2000) has persuasively argued that our current worldview is still heavily biased by Neo-Platonist metaphysics. Opposition of scientists and non-scientists alike to a recent paradigm shift in the cognitive sciences, that our mind is shaped by the evolutionary process of natural selection, is at least partly founded on a top-down approach, even though they themselves may not be aware of that. A bottom-up approach consists of a metaphysical view where lower faculties shape and influence higher faculties, and where complex forms arise from simpler phenomena. A standard example is Darwin's theory of natural selection: life arose from simple physical processes, and complexity in life arose gradually due to a blind process of natural selection (Darwin, 1859 [1985]). Living simple organisms arose from lifeless matter, living complex organisms evolved out of simpler organisms.

Which metaphysical framework is best suited for an inquiry into the foundations of human mathematical competence? There are several good reasons why a bottom-up approach is epistemologically more valid: it is refutable, more parsimonious, and has a greater explanatory power than the top-down approach. A top-down approach is irrefutable, indeed impossible to test. If we cannot exert any influence on the alleged higher faculties that influence us, then we cannot hope to study them on a scientific basis. On the contrary, a bottom-up approach usually relies on simple things to explain complex things. These can be measured scientifically, e.g. under the microscope, and processed statistically; in other words, the data are quantifiable. The validity of a model also depends on its *explanatory power*. A model that provides a coherent explanation for a range of phenomena that were unexplained before, or explained by unrelated models, deserves attention. Related to explanatory power is *parsimony*, which means explaining complex phenomena with simple explanatory mechanisms. Parsimonious explanations are favoured over rich explanations, which are usually unwarranted presuppositions. Although Occam's razor cannot be used indiscriminately to decide between competing theories, it has in the past repeatedly proven to be a reliable tool. Thus, Copernicus' model of the planets orbiting around the Sun was a simpler explanation for observed trajectories of the planets than the complex medieval models of the Sun and planets revolving around the Earth. Top-down approaches offer complex, rich explanations for complex phenomena, while often relying on unwarranted presuppositions. Bottom-up theories explain complex phenomena in terms of less complex ones, and are therefore more elegant as well. Finally, a scientific model is more likely if it provides a good *fit* to the observed facts. Since top-down theories often rely on unobservable or immeasurable higher faculties, they often fail to provide a good *fit* to reality. Bottom-up theories can or cannot provide this *fit*, but they can at least be tested using quantitative methods.

1.2. *Top-down and Bottom-up Approaches in the Philosophy of Mathematics*

In the philosophy of mathematics, two schools stand out as models for our top-down/bottom-up categorization. So called Platonists believe that mathematical objects have their own existence, outside the human mind. When new mathematical ideas arise, these are not *invented* by mathematicians – since they were already out there – but merely *discovered*. How the link between this world of mathematical ideas and the brain of the mathematician is established, remains vague and unexplained. This connection is empirically unobservable. The Platonist view on mathematics is a top-down approach. A complex phenomenon (mathematics) is explained by an even more complex one (an unobservable Platonic world). We will never be able to study this world on a scientific basis, which makes the Platonist idea irrefutable. The notion of a world of mathematical ideas seems incredibly rich and unwarranted. Its only advantage might be that it seems to provide a good fit with the fact that some mathematical ideas have unusual and surprising side effects (even to their discoverers), e.g. game theory has become an indispensable tool in economics, evolutionary biology, and anthropology.

On the other hand, constructivism provides an example of a bottom-up approach to mathematics. In this approach, mathematical objects do not exist outside culture or the human mind. Humans construct mathematical ideas, which are part of a cultural tradition (White, 1949, 282–302). Constructivism is testable, refutable, and does not rely on rich explanations. It provides a good fit to the history of mathematics, as exemplified in the evolution of π. Egyptian, Babylonian, Chinese, and Indian mathematicians expressed the relation of the radius of a circle to its circumference by a numerical. Over time, the numerical expression became more and more accurate in describing this relationship (Gheverghese Joseph, 1990, 190–191). Despite its epistemic simplicity, there are several methodological drawbacks to the constructivist approach: it leaves unexplained the surprising applicability of mathematics to empirically observed facts. We still lack a scientific model for a bottom-up approach of mathematics that meets the criteria of testability, explanatory power, and fit with observed phenomena. In this paper, I will try to outline what this model should look like, and what its epistemological framework should be.

1.3. *A Brief History of Western Notions About the Human Mind*

Humans have always had a desire to set themselves into context, both as individuals and as a species. This desire undeniably stems from the scope and properties of the human mind. One of the greatest challenges to science is to investigate how this mind came into being (Gowlett, 1984, 167). It is therefore not surprising that the mind itself has been the object of scientific research and philosophical reflection for centuries.

Traditionally, the western notion of the human mind has been a top-down approach. This is largely because Christianity has dominated western thought since late Antiquity. In Christian metaphysics, following Plato, a human being consists of a body and a soul. Furthermore, Descartes' view on human cognition has had a great influence on our way of thinking about human cognitive abilities. The Cartesian distinction between body and soul made every biological approach to the human mind impossible, because the biology of our body is perceived as completely separate from the workings of the mind. Locke's doctrine of the mind as a blank slate states that it is infinitely plastic, with all its structures coming from impressions, learning, experience and socialization. Although the empiricist approach was not the only view on the human mind, it has dominated Western scientific notions on the mind until the second half of the twentieth century. Some nineteenth century scientists questioned the blank slate view (e.g. Freud), but by the first half of the twentieth century it was the prevailing view. In the social sciences, anthropologist Franz Boas showed that cultural change did not depend upon any evolutionary or biological basis. By the 1920s and the 1930s, the evolutionary approach to human mind and culture was eradicated (Foley, 1995 [1997], 4). Anthropologists in the field (e.g. Malinowski, Mead) found support for this idea in non-western cultures, which, according to them, exhibited an unlimited variability of human behaviour and social patterns. Recent re-examination of their results indicates that they often 'nudged', or in the infamous case of Margaret Mead, even forged their data to make them more in keeping with the blank slate theory (Freeman, 1996). This paradigm is also apparent in other disciplines, e.g. in developmental psychology, where Piaget's theory of staged cognitive development was generally accepted as an accurate account of the way in which the child's mind developed. According to him, children are born without knowledge and gradually learn to deal with the world as they are confronted with its properties. In the field of research on animal cognition, a similar pattern can be observed. In the late nineteenth century, it was generally believed that animals had thoughts and emotions not unlike our own. Darwin's *Expression of the Emotions in Man and Animals* (1872 [1998]) exemplifies this. This idea was later challenged by behaviourists, who explained every action an animal performs as the reflex (response) to a stimulus. Every response thus depended on previous experiences. Take Pavlov's dogs: they were taught that food always comes after an auditory signal. After a while, the dogs started salivating at the signal, even without any food (Cziko, 1995, 88–89). In the behaviourist model, like in the blank slate model for human cognition, the animal mind is a blank slate, shaped by experience alone.

In the second half of the twentieth century, scientists of various disciplines started to doubt the blank slate model of human cognition.Chomsky (1972) noted that children learn language much faster than they are supposed to, considering the poverty of the stimulus. They infer language rules (verb conjugation, word order) that are never explicated by their parents from a relatively limited sample of spoken language. They can make new sentences they have never heard before according to those rules. The only solution to this riddle is that the human mind has information to add to the stimuli from the outside world. In other words, it has a device to decode rules from the language the child hears when it learns to speak. Evolutionary epistemology provided a philosophical background for these nativist ideas. Campbell (1974) proposed that the brain, like other organs, is the result of natural selection, and that this process shapes both our perception and cognition. Imagine an animal whose perception corresponds to its environment. If it sees a predator approaching, it correctly judges it a predator, and flees. Imagine another animal with an unreliable set of perceptual abilities: it might fail to interpret the predator as such, and will be eaten. Undoubtedly, the animal with a reliable perception can reproduce better than the one with an unreliable perception. Our cognition and perception do not deceive us, because it is vitally important that they correspond to the world we live in. We would never have survived in a hostile world if our view of the world did not have some correspondence with the world. Since the 1970s, cognitive scientists from various disciplines like developmental psychology and cognitive ethology started to challenge the blank slate view on the human mind (see e.g. Barkow et al., 1992; Hirschfeld and Gelman, 1994; Shettleworth, 1998). Space precludes an exhaustive description of these paradigm-shifts; instead, I will focus on their impact on the research on human mathematical abilities.

2. A STARTING POINT TO A DARWINIAN APPROACH TO MATHEMATICS: MATHEMATICAL ABILITIES OF ANIMALS

All biological inquiries into the nature of human cognitive abilities have to account for the apparent gap between human cognitive abilities and those of other animals (Gowlett, 1984, 167). Humans possess a range of unique faculties, e.g. complex language, art, mathematical abilities, which are absent in other species – even in our closest living relatives the African apes. How can this be explained? A good starting point to a biological approach to human cognitive abilities is to try to establish how animal minds work in general. In order to do this, we must turn to cognitive ethology, the science of animal cognition. Cognition refers to the mechanisms by which animals acquire, process, store and act on information from the environment. These include perception, learning, memory and decision-making. Cognitive ethology is concerned with how animals process information, starting with how this is acquired by the senses (Shettleworth, 1998, 5–6). Research into the cognitive abilities of animals mainly has been conducted for two reasons: first, to gain a better understanding of the mental capacities of humans by testing other animals for these capacities, and second, as an end in itself, an inquiry into the cognitive abilities of different species, rather than an attempt to understand our own species better. We can attain a better understanding of cognitive abilities of our own species, when we compare them with those of other animals (Davis and Pérusse, 1988, 561).

2.1. *Clever Hans and Morgan's Canon*

The story of Wilhelm von Osten and his horse Hans is infamous in cognitive ethological research. He claimed he had taught his horse to perform arithmetic. When presented with a simple addition or subtraction exercise written out on a chalkboard, Hans would tap the correct result with its hoof. Hans always managed to do this, and his master firmly believed in the horse's mathematical skills. In 1904, a committee of university experts investigated the matter. It turned that Hans was very clever indeed, though he was more of a psychologist than a mathematician. He relied on clues his master and attending audiences unconsciously gave him: the tension in his master and the audience peeked as he was close to the correct answer. When this clue was not available – if no one in his proximity looked at the blackboard, or knew the answer – he failed to give the correct answer. This story infamously illustrates the methodological difficulties researchers of animal cognition face (Devlin, 2000, 24).

There are two solutions to the Clever Hans problem. The first is an epistemological tool, Morgan's Canon. According to this canon, one must explain an animal's behaviour in the simplest possible terms of cognitive capabilities (Cartwright, 2000, 5). It is a special case of Occam's razor. For instance, if bees make hexagonal honeycombs, we could suppose that bees possess innate capabilities for geometry. The honeycomb conjecture, which holds that a hexagonal grid represents the best way to divide a surface into regions of equal area with the least total perimeter, was recently proved by Hales (2002). Does the bee consciously construct hexagons? It seems more parsimonious to suppose that it instinctively uses several short cognitive circuits to construct a honeycomb, e.g. the angles (120°) can be constructed, with gravity as a point of reference, because a bee has a special organ in its neck, which tells it the direction of the gravitational pull. When a honeycomb is damaged, bees fix it using irregular polyhedral shapes instead of neat hexagons (Ball, 1999, 25–27, 48–49). The second solution to the problem posed by Clever Hans is rigorous experimenting, in which all possible interfering variables are eliminated. The evolutionary approach (e.g. Shettleworth, 1998; Shettleworth, 2001; Hauser, 2001) is also very useful in predicting which cognitive skills will arise, and why. According to this approach, the question "Can animals do geometry?" is unjustified, since it rests on an anthropocentric view on the world, which puts humans forward as the point of reference. Instead, we should ask ourselves: given the specific ecological conditions, how can this animal find its way? By using this approach, researchers have been able to narrow down the choice of their subjects (e.g. seed-storing birds are supposed to have better spatial cognition than non-seed-storing birds).

2.2. *Evidence of Animal Mathematical Cognition*

Cognitive ethologists differ from behaviourists in their claim that animals have intentions and beliefs, and that they consciously reflect about their plans and course of action. Cognitive ethologists do not deny the possibility that animals can have complex thought processes (Shettleworth, 1998, 7). Despite this epistemological framework, current testing of animal's mathematical cognition is conducted in a rigorous way, to minimize the risk of the Clever Hans phenomenon. Morgan's Canon is always applied to test if a simpler explanation could account for the results. It often proves difficult to set up an experiment in which an intuitively appealing idea (e.g. animals have at least a rudimentary sense of number) can be tested, and in which all possible more parsimonious explanations (e.g. animals are influenced by the behaviour of the researcher) can be excluded. Nevertheless, cognitive ethological research has provided us with a wide variety of evidence that animals can perform mathematical tasks, among them an understanding of number (numerosity), geometry, and algorithmic behaviour.

2.2.1. *Numerosity*

The story of Clever Hans may have cast a pall over research on animal counting, but did not end it. Research on animal numerical abilities is important for issues in animal cognition research, and in current debates about animals' cognitive abilities, e.g. behaviourism vs. cognitivism, skepticism vs. credulity. Whereas earlier research was restricted to evidence of explicit counting (which animals rarely, if ever do), current research focuses on a wider variety of internal mathematical operations involving number (Shettleworth, 1998, 364–365), like the sensitivity to ratios of time intervals, estimation, subitizing (to quantify small numbers of items ($n < 4$) without conscious counting), relative numerical judgments (which quantity is bigger), and estimation. These numerical skills are generally described as *numerical competence* (Davis and Pérusse, 1988). Tests on animal numerical competence have been conducted in the wild and in laboratories. Observations of a wide variety of animals in the wild suggest that they engage in forms of calculation to maximize energetic intake on foraging trips. They calculate average rates of return in one patch of food so that they can either stay or switch to another patch (Emlen, 1966). Chimpanzees in the wild sometimes attack and kill a member of another group. They only attack the individual if he is alone, and if the attackers are at least three in number (Hauser, 2001, 55).

The earliest laboratory experiments on animal numerical competence were conducted by Köhler in the 1950s and 1960s (Devlin, 2000, 22). Jackdaws were trained to select from a number of covered boxes the one having a certain number of spots on the lid (ranging from 2 to 7). Other experiments in the 1980s (cf. Shettleworth, 1998) have shown that rats and pigeons can be taught to respond selectively to different numbers of objects. However, this is an unnatural sort of behaviour that can only be elicited by relatively stressful experimental conditions (Griffin, 2001, 132). To overcome this problem, the following experiment (McComb et al., 1994) was conducted with free-ranging lions in the Serengeti Plain (Tanzania). The experimenters taped roars that were unfamiliar to the pride they were studying (lions can distinguish roars from familiar and unfamiliar individuals). When the tape with only one unfamiliar roaring lion was played, the lions of the pride were more likely to approach than when three unfamiliar roaring lions were heard. This experiment implies that animals can also count non-visual stimuli. However, this does not rule out that lions can make a rough estimate of the number of lions according to the intensity and sound-pattern of the roaring, instead of actually counting them.

Another experiment in natural conditions shows that peacocks can perform number estimation. A peacock displays its tail to seduce females into mating with him. Peahens tend to choose males with tails displaying more eye-shaped spots over those with less eye-shaped spots. The result is that peacocks have great differences in reproductive success, which is why (according to sexual selection theory) peacocks evolved spectacular tails with a great number of eyespots. To test whether this observation actually means that females make relative numerical judgments, or whether the males with more eye-spots are more desirable for other reasons (e.g. better health), an experiment was conducted in which some males were captured by researches and had 20 eye-spots removed at the end of a mating season. Control males were captured and handled in a similar fashion, but did not have any spots removed. Peacocks with removed eye-spots had less mating opportunities during the next mating season, whereas the control males that had been similarly handled (though without having any eye-spots removed) did not. This implies that peahens use number estimation to choose between potential mates (Petrie et al., 1991).

Chimpanzees are our closest living relatives. Numerical competence in chimpanzees might therefore be important to our understanding of human numerical competence. The female chimpanzee Ai was trained in the Primate Research Institute at Kyoto University to use symbolic language (on a computer panel) to test her numerical skills. This research focused on the cardinal and ordinal meanings of number. She was extensively trained to label objects according to colour (red, green, yellow, blue or black), to object recognition, and to number (1–9). Ai typed the answers on a keyboard. Gradually her accuracy in number labelling of novel samples improved as her training progressed to include ever larger numbers and more objects and colours. In a typical test, Ai sees five red toothbrushes. She presses on the appropriate keys 'red', 'toothbrush' and '5'. Ai could freely choose in which order she assigned the attributes, but she consistently chose number as the last attribute. In addition, her accuracy was lower in number-attribution than in colour-attribution or object-recognition. This test does not show whether Ai experiences difficulties in counting, or whether she has difficulties assigning a number (symbol) to a number of objects. In an additional study on cardinality, Ai was trained to count the number of dots on a screen – that is, to assign them to a numerical symbol on the keyboard. Her performance was highly accurate, but not perfect (83.6 % for randomly positioned dots, 79.2% for dots arranged in a single line). The time required to complete this task was compared to that of human subjects. Chimpanzees, like humans, take remarkably less time to count dots where $n < 4$ than those where $n > 4$. This unconscious swift counting mechanism is called subitizing. The results show that chimpanzees do not seem to count the dots consciously (sequential tagging, as humans do), but instead use a form of estimation when the number is larger than 4 (Biro and Matsuzawa, 2001). Rumbaugh et al. (1987) tested chimpanzees' ability to do mathematical operations (summation). Chimpanzees were presented with two trays, each containing a pair of food wells with chocolate bits. Each tray held a different total number of chocolate bits. E.g. the left hand tray contained a well with one bit and one with three bits, whereas the right hand tray contained a well with two and one with four bits. Chimpanzees gradually learned to pick the greater sum, especially if the ratio of larger to smaller amounts was relatively high. Control tests showed that chimpanzees do not base their choice on the contents of just one well, but on the sum of the bits in both wells.

2.2.2. *Geometry*

Many animals regularly return to particular locations (e.g. hives, nests, caches), which implies that they possess so called cognitive maps, information to guide them from their current location to their goal. Do these cognitive maps contain information about the geometric relationship among objects? A wide variety of animals with a relatively primitive brain can find their way, e.g. bees forage routinely between their hive and feeding sites hundreds of meters away. Both tests in laboratories and field observations show that bees gradually get familiar with the hive and its surroundings (Shettleworth, 1998, 311). Does this imply that social insects possess cognitive maps? Judd and Collett (1998, 710) have proposed a more parsimonious explanation. They have shown that ants store multiple views of the position of landmarks retinotopically to find their way from a newly found food source back to their nest, and all the way back again. They do not make an abstract representation of the geometric relationships between landmarks. Similarly, rats do not seem to consider geometric relationships when they navigate inside a maze (Shettleworth, 1998, 313). However, recent experiments have shown that some species, under some conditions, use geometry to determine their orientation, to identify landmarks, or to find a place. This implies that the animal identifies a landmark not by its appearance, but by its spatial relationship to other landmarks (Bielger et al., 1999). In one experiment (Kamil and Jones, 1997), Clark's nutcrackers (a species of corvid) were encouraged to find the halfway point between two landmarks. The learning process involved a seed which was partially (but still visibly) buried halfway between two plastic pipes. On the test trial, the seed was entirely buried. After a while, the birds were presented with new distances between the pipes to test their ability to generalize the geometric relationship between the seed and the two landmarks. The birds readily learned to bisect the interlandmark distance. They correctly found the halfway point when the landmarks were presented with new distances between them. The geometric sense of the birds was quite abstract, since alteration of the height of one of the landmarks did not affect their success at finding the seed. Simpler explanations were ruled out, e.g. finding the seeds by smell or by cues other than geometry. It might be argued that the birds, during the training session, learned the specific vectors for each interlandmark distance during training. In this view, during a test, they would average the vectors of the interlandmark in the training session closest to the novel test (Bielger et al., 1999). This seems unlikely, since the birds are more accurate in finding the line connecting the landmarks than in locating the correct position on the line. This suggests that they are making two separate decisions (Kamil and Jones, 1999): finding the line connecting the landmarks, and determining the halfway point on it. Clark's nutcrackers store up to 33,000 pine seeds in literally thousands of cache sites during the autumn months. The seeds constitute the bulk of their winter diet, and are subsequently fed to their nestlings in spring (Gould-Beierle and Kamil, 1999). It is therefore not surprising that these birds use visual cues and geometric algorithms to find the stored seeds back. Research on marsh tits (a seed-storing bird, not closely related to Clark's nutcracker) yielded similar results. They were allowed to store seeds, which they had to take from a central place to different cache sites of their own choice. The order in which they eventually retrieved the seeds was different from the one in which they had originally stored them. Moreover, they did not retrace the same paths they had taken while storing them (Shettleworth, 1998, 315). Preliminary field observations and laboratory-tests on primates (especially chimpanzees and vervet monkeys) indicate that primates also use geometric relationships in making a cognitive map (Shettleworth, 1998, 317).

2.2.3. *Algorithmic behaviour*

Animals often give evidence of algorithmic behaviour while systematically searching for food, since food is often patchily distributed among locations that vary spatially and temporally in profitability. In one experiment, two species of intertidal fish (the 15-spined stickleback, and the corkwing wrasse) were tested on their use of learned patterns of movement (algorithmic behaviour) in foraging strategies (Huges and Blight, 1999). An eight-arm radial maze was designed, with a central compartment, which gives access to the arms, with one food cup placed at the end of each arm. Each fish was transferred to the central chamber and allowed to forage freely inside the maze, where it could eat the food-items it encountered. Each food cup contained a single shrimp. To forage efficiently, subjects had to avoid arms that were already depleted within the trial session. In absence of any spatial cues (all the arms looked identical), the fish improved their foraging efficiency by spontaneously developing the algorithm of visiting every third arm. Algorithmic behaviour is especially efficient when no sensory cues provide information to the location of food-items. Thus, when coloured tiles were put in arms of the maze, their previous algorithmic behaviour was largely subsumed by the use of spatial memory, i.e. the coloured tiles were used as landmarks.

3. HUMAN MATHEMATICAL CAPABILITIES SITUATED IN HUMAN BIOLOGY: THE BRAIN AND MATHEMATICAL COGNITION

3.1. *Mathematics as Innate Behaviour?*

Behaviourist psychologists and anthropologists thought of the human brain as a general purpose computing device, processing information from the outside world. In this view, the responses a brain produces are shaped by experience alone. The work of Piaget (1952) provides a good example for this view on the brain in relation to mathematics. According to him, children gradually learn how to deal with both physical objects and mathematical reasoning as they get older. The ability to count, he thought, arises gradually at the age of five. It requires the prior development of logic skills such as transitive reasoning and putting two sets of objects (words for numbers and the objects to be counted) in a one-to-one (ordinal) correspondence, and an understanding that the number assigned to the last object represents the total number of objects counted (cardinality). In this intuitively appealing conception of the mind mathematics represents a 'higher', 'more abstract' form of reasoning, which young children do not possess (Gelman and Brenneman, 1994, 372–373).

Piaget has been challenged by developmental psychologists, who have investigated the numerical reasoning principles of young infants. Starkey and Cooper (1980) presented infants (6–12 months old) with slides with differently arranged sets of dots, ranging from one to four. They proved that infants could tell the difference between sets of one, two or three dots. They accustomed them to watch slides with the same number of dots, but in different arrangements. When they showed a slide with a different number of dots, the infants stared longer at that slide – which indicates that they show more attention to the number of dots than to their arrangements. The infants could also discriminate among small arrays that were identical in length, but not in number (e.g. 2 widely spaced dots vs. 3 more densely spaced dots). When the number of dots exceeded 4, the infants could not tell the difference between, for instance, 4 and 6, and alteration in the number of dots in this case did not affect the time they looked at the slide. This use of larger numbers of dots ruled out the possibility that they used other cues than number (such as overall brightness of the image) in discriminating between the slides. The spontaneous and automatic counting of small collections of objects ($n < 4$) is called *subitization*.

In an experiment by Starkey et al. (1983) seven-month-old infants were shown pairs of slides with either a number of dots or a number of objects (ranging from 1 to 4), varying in colour, shape, size and arrangement. At the same time, they heard a number of drumbeats (ranging from 1 to 4). The infants preferred – i.e. looked significantly longer at – slides with the number of dots or objects which matched the number of drumbeats they heard. This indicates that infants have an innate sense of number, which is unrelated to the modality (visual or auditory) and type (object or event) presented.

Wynn (1992) tested five-month-old infants' ability to add and subtract small numbers ($n < 4$). A typical experiment runs like this: the infant is shown a stage with one puppet (its mother holds the infant, but the position of the mother is such that she cannot observe the stage). The curtain drops. The experimenter visible puts another puppet behind the curtain, which is then lifted. Some infants get to see two puppets, a number they should expect. Others however, only see one puppet (the other being secretly removed while the curtain was down). The experiment shows that infants who see the impossible result ($1 + 1 = 1$) stare at the screen for a significantly longer period of time than those who see the expected result ($1 + 1 = 2$). According to Wynn (1992, 749), this test indicates that subitizing is not a holistic recognition of non-numerical patterns, but that it encodes ordinal relationships between numbers.

These tests provide sufficient proof to falsify Piaget's view on human mathematical skills. Apparently, simple mathematical exercises (counting, addition, subtraction) are not the result of abstract, complex reasoning, but are innate – since parents could not have taught these young infants how to perform these skills. We have seen that other animals have the same biologically determined ability to attend to small numbers of objects or events in their environment (Dehaene, 2000, 987). It is necessary to understand how these abilities are situated in the human brain.

3.2. *Mass Modularity*

The cognitive psychologists realized that human perception is not a 'dumb'system that is triggered by outside stimuli like a simple reflex, but a 'smart' one: it filters out irrelevant information, and conversely, it can produce a lot of inferences on the basis of very poor stimuli (e.g. even the most rudimentary drawing of a face produces a rich set of inferences, like emotional state, gender, or age). Fodor (1983, 1985) tried to reconcile the paradoxical facts that our perceptual system is apparently 'smart' and 'fast', behaving both like a reflex and like a sophisticated processor. He concluded that our brain contains specialized systems to process specific kinds of input from the outside world. He called these *modules*. Modules are separate from each other and from other cognitive processes. They work in isolation, only dealing with input and knowledge in which they are specialized, e.g. a language module will not deal with colors or shapes. Good support for the modularity hypothesis comes from patients with brain damage exhibiting domain-specific pathologies: they are bad in one specific domain, while their other mental faculties remain unaffected, e.g. cases of aphasia, the inability to recognize faces, and numerous cases of *acalculia* (the inability to count or perform arithmetic). Modules are fast, because they are specialized. This makes evolutionary sense: an animal that could recognize a predator or prey quickly greatly increases its chances to survive and reproduce. Modules are also 'smart', because the environment in which they act is complex and variable.

Unfortunately, Fodor does not provide a good theoretical framework to explain innate human mathematical abilities. The title of his book, '*The Modularity of Mind*', is misleading: perception according to him is modular, whereas cognition is non-modular. According to him the cognitive processes are 'deeply mysterious' and will never be explained by scientific research. Sperber (1996) proposed a more radical modular approach: every single thought process, perceptual thinking as well as conceptual thinking, and even meta-conception (thinking about thinking, e.g. he thinks that I think about ...) is modular. His approach envisages a mass of modules. Hirschfeld and Gelman (1994) provide a wealth of experimental evidence that people have specialized conceptual modules to deal with specific kinds of information, e.g. information about people, about living kinds, and about physical laws. The conceptual modules work with information provided by perceptual modules, or by other conceptual modules. Innate physics provides a good example. People have specialized cognitive circuitry to deal with the physical properties of inert objects. Even very small infants show surprise when they are confronted with physical events that violate the laws of physics, e.g. when two solid objects do not collide but gently float through one another. Information for this module is provided by diverse visual modules (e.g. a module dealing with movement, one with shape, etc.). A possible flaw in this line of reasoning is, that some human activities are very recent in human history, e.g. chess, mathematical theory, formal logic, and it seems unlikely that specialized cognitive circuitry should have arisen for them in such a short span of time. However, according to Sperber (1996, 134–143), a module does not only work on the domain for which it has evolved (its proper domain). It can also be triggered by stimuli that resemble the proper domain (its actual domain). The only thing it can do is respond to stimuli with certain properties for which it has been selected by natural selection. This makes sense, because a module does not 'know' for which domain it has been evolved. Imaginary beings like unicorns, dragons, angels provide an example. We entertain ideas about these creatures and store information about them in a module for living beings. We might for instance use modules for navigation in space (which we normally use in our day-to-day locomotion) when we play chess. We may now ask ourselves: is mathematics a domain for which a specialized module exists, or is it part of the actual domain of a module which has arisen for a different proper domain, e.g. like language?

3.3. *Domain-specific Pathologies Related to Mathematical Abilities*

If you want to know what a particular part of a machine is for, the best way to find out is to remove the part and see how (and if) the machine works without it. Conversely, if a part of a machine doesn't work, and there is a flaw in the working of the machine, it can tell you what that part is for. Butterworth (1999a) has done extensive research on domain-specific pathologies related to mathematical abilities. The patients he examined all had suffered some kind of brain damage (e.g. a stroke), affecting some of their cognitive skills while leaving others unaffected. A stroke is an interesting pathology, since it damages only the area of the brain fed by the affected blood vessel, thus inducing only domain specific damage. Most of the patients exhibiting *acalculia* (the inability to count or do arithmetic) were affected in the left parietal lobe (the area roughly above and behind one's left ear). One patient recovered fully from a stroke – she could still talk, reason abstractly (if x is taller than y, and y is taller than z, is x taller than z?), had good memory, but could neither count, add, nor subtract. She could not subitize (tell at a glance) how many dots were presented to her on a card, even if there were less than four, as we have seen, something even infants can manage. Even the simples arithmetic is more than just fact retrieval. For example, two patients could perform multiplication-tests quite well (probably because the tables of multiplication were verbally stored in their memory), but were unable to solve 12×4 given that $4 \times 12 = 48$ (Butterworth, 1999b). However, to prove that a certain cognitive skill can really be assigned to one or more modules, we need two kinds of evidence: a patient in whom this particular skill is missing, while other skills are still intact, but also one who still exhibits this skill, but has lost some other cognitive abilities. Finding this evidence is important, since many cognitive scientists (e.g. Chomsky) believe mathematics is just a special case of language, and thus processed by the language module. If no subjects could be found, the hypothesis that mathematical reasoning differs from linguistic skills would be untestable. Compare this situation to someone who has an injured knee, who can no longer run, but still manages to walk. If we would assume from these data that a knee is used in running, but not in walking, this would be an erroneous conclusion. Unfortunately, since language is also situated in the left part of the neocortex (the great bulky mass of grey matter that constitutes the largest part of the human brain), most patients who lose this skill also lose mathematical skills. One patient, affected by Pick's disease (a disease causing dementia, similar to Alzheimer) lost almost all his linguistic skills, but was still good at addition, subtraction and multiplication exercises presented to him on paper (Butterworth, 1999a, 163–169). The data suggests that humans possess a mental device which deals with recognizing small collections of objects ($n < 4$) at a glance, deals with calculation and with comparison of quantities (which is the bigger collection of a given set of objects).

Normal subjects can equally provide evidence for quick, domain-specific handling of mathematical problems. Adults can quickly and without uncertainty tell which of two digits is larger. How is this accomplished? It might be that the judgment is based on a less perceptual, cognitive level, like memory access in which the subject retrieves the numerals from memory and compares them. Moyer and Landauer (1967) have examined the time required for relative numerous judgments. Subjects were presented with pairs of numerals between 1 and 9. Each digit appeared 24 times to the right, and 24 times to the left in a random order. Subjects were instructed to press either the left-hand or right-hand of two switches according to whether the left or the right digit was the larger. Surprisingly, the more the stimuli differed, the quicker the reaction was – the *distance effect*. There was no correlation between the difference in reaction time and the shape of the numerals, e.g. 7 and 8 differ much in shape, but took a longer time than 3 and 8, which are much more similar in shape. The cognitive mechanism for this task could not be purely attributed to memory access, since these relations are hard to explain by direct memory look-up. Later tests confirm this early experiment, regardless of whether the stimuli were presented as Arabic numerals or as random dot-patterns. The test has also been conducted with monkeys. The distance effect turned out to be strikingly similar in monkeys and humans (Brannon, 2003, 279). When Moyer and Landauer (1967) conducted this experiment, there was no epistemological framework to explain the results. Now we have the modular approach. If number is indeed processed by a specialized number-module, it is not surprising that this module is more easily activated when the difference between two stimuli is large. Similar experiments with pairs of pitches or pairs of colours yielded similar results: the greater the difference between two stimuli, the smaller reaction time tends to be (Moyer and Landauer, 1967). If adults represent number non-verbally with similar abstraction i.e. if mental magnitudes are indeed modality-independent, we can safely assume that number is processed by a specialized conceptual module. Barth et al. (2003) asked subjects to indicate whether two series of events (flashing circles or a series of tones) carried the same number of events. Results indicate that there was no cost in accuracy whether the test was crossmodal (e.g. a series of flashes plus one of tones) or intramodal (e.g. two series of tones). This test provides evidence for the idea that mathematics is a cognitive domain that is processed differently by our brain than for instance, physics or psychology. This *number module* can rapidly perceive and count small collections of objects (subitizing), and it can make rough estimations of larger quantities. Some might even argue that these tasks are handled by two different modules (Carey and Spelke, 1994, 176).

3.4. *Direct Evidence for Domain-specificity in Mathematical Skills*

The most direct evidence we can obtain for any claim for domain-specificity of a certain skill is to examine the brain itself, and to look which parts of the brain are active during a specific task. If all subjects use the same part(s) of their brain to perform the same task, we can infer domain-specificity for that task. Fortunately, the brain can be studied directly using *functional magnetic resonance imaging* (fMRI) or *Positron Emission Tomography* (PET). Both methods rely on the fact that regions of the brain that are more active during a task require more energy in the form of glucose and oxygen (Greenfield, 1998, 35–36). Both can provide highly detailed maps of brain activity. The evidence for domain-specificity is increasing as the number of PET-scan and fMRI studies in which subjects perform a wide variety of tasks increase, like recognition of animals or tools (Martin et al., 1996), and recently, mathematical tasks (Dehaene et al., 1999; Dehaene, 2000). Using fMRI, Dehaene et al. (1999) have identified increased activities in the intraparietal sulci of both hemispheres when subjects performed exact and approximate number tasks. They showed more activation in the approximate number tasks than in the exact ones, maybe because rote-learning (memory) plays a larger part in exact number tasks. In the close vicinity of these areas are neural circuits that control finger movements and eye-movements. This could explain why finger counting and finger calculation is universally practised, and why it is an almost universal stage in the learning of exact arithmetic in children (Butterworth, 1999b). Areas for mental rotation and attention orienting are also close by (Dehaene et al., 1999, 970–971), which might account for the fact that visuo-spatial representation (e.g. in the case of geometry) and number tasks are closely linked in mathematics. Dehaene (2000, 994) made a PET-scan study of subjects performing multiplication and comparison of number pairs. He found bilateral parietal activation, i.e. in the left and the right hemisphere parietal lobes, confined to the intraparietal region. Interestingly, the strength and duration of these activities increased as the tasks got more difficult, but did not depend on the modality or notation in which the numbers were presented. E.g. in a comparison task for number pairs similar to that of Moyer and Landauer (1967), close distance between the numbers activated the mathematical modules more than greater distances. Again, notation did not matter in this task, since '7' does not resemble '8' any more than '2' resembles '8' (Dehaene, 2000, 995).

4. ADAPTATIONISM AND HUMAN MATHEMATICAL ABILITIES

How did our mental faculties come about? Why do we share some mathematical skills (e.g. numerosity, geometry, algorithmic behaviour) with other animals, and why are others uniquely human? Behavioural and cognitive traits in animals (and humans) can be explained in several ways (e.g. behaviourism, cognitive psychology, behavioural ecology). Evolutionary theory provides a strong deductive framework for explaining human and other animals' cognitive capacities. One of the central achievements of modern evolutionary biology has been the recognition that selection operates on the level of the gene (Williams, 1966; Tooby and DeVore, 1987). Genes govern the development of organisms. The morphology, development and behaviour of an organism are an expression of its genes. These genes are copied from one generation to the next when the organism reproduces itself. This copying process is not flawless: small mistakes occur, resulting in variation in the gene pool of a species. Every organism has the ability to reproduce itself in very large numbers, but we observe relatively stable populations. This is because most resources (food, water, space) are limited. Some variations of a gene give rise to traits which give its bearer a reproductive advantage over other members of the same ecological community, e.g. it can be beneficial in feeding strategies (increasing the likelihood that the bearer will survive), or it can make its bearer attractive to members of the other sex (increasing likelihood that its bearer will reproduce).

These genes have a greater chance of being passed on to the next generation. In other words, natural selection favours genes that increase the chance of reproduction of an organism or increase its *inclusive fitness*. Thus, a beneficial gene will be passed on to the next generation and will spread in the population over time, while a deleterious one will die out, because organisms bearing them cannot reproduce as well. Thus, any complex heritable trait can be expected to be *adaptive,* i.e. to be beneficial for its bearer's reproductive success. The adaptationist approach holds that natural selection is the only important force driving evolutionary change (see e.g. Orzack and Sober, 2001). Of course, random genetic drift can also account for many genetically coded features. This explanation, however, is less applicable for traits that exhibit *complexity* and *apparent design*. This is because the accumulated probability that a large number of genes cooperate to make a complex and ordered trait is extremely low (Tooby and DeVore, 1987, 194–195). As we have seen, the cognitive circuits governing mathematical abilities in humans and other animals are extremely complex, fine-tuned and accurate. The most likely explanation therefore, is that these are the result of natural selection, not of the stochastic effects of genetic drift.

The brain is an organ that takes decisions that ensure survival and reproduction, e.g. about mate choice, food acquisition, predator avoidance. Because animals live in different natural and social environments, these organs will differ in their internal organizations according to these environments. As the brain is a costly organ (requiring a vast amount of energy compared to other organs), every mental ability an animal exhibits, has to serve some evolutionary purpose. These simple facts of evolutionary theory predict that every animal will possess a brain that is adapted to its environment, i.e. the animals it must compete with (conspecific and non-conspecific), organisms it eats or by which it gets eaten, and climatological and other physical factors. A brain can be compared to a toolkit: every animal has a toolkit appropriate to its survival and reproduction. Specific problems give rise to specialized cognitive solutions, e.g. animals living in complex social groups will have cognitive mechanisms to govern social interactions which solitary animals will lack (Hauser, 2001). From an evolutionary perspective, everything a brain can do has contributed to the *fitness* of the ancestors of the bearer of the brain. From an anthropomorphic Platonist point of view cognitive thought-processes are more 'advanced' than perceptual processes. This distinction is ungrounded. There is no clear boundary between perception and cognition, since even the most simple perceptual task requires computation (e.g. a frog cannot perceive anything, unless it moves). Moreover, from an evolutionary point of view, perception is no less vital than pure cognition (whatever that may be). Not many animals have invested as much as humans in cognitive modules; on the contrary, most animals possess a fairly simple brain with fewer neural connections. A brain is not only an interpreter of sensory input, it is also vitally important in decision making about feeding, finding a mate, avoiding risks, care for offspring, forging social alliances, and so forth. Divergence from the universal toolkit occurs when species confront unique ecological or social problems (Hauser, 2001, xvi).

4.1. *Mathematical Abilities We Share With Other Animals*

The research on animal and human mathematical abilities I have discussed indicates that mathematical abilities are widespread among the mental toolkits of different species. This can be the result of two scenarios: either mathematical abilities arose once, in a common ancestor to all the animals currently possessing the trait, or it could have arisen several times independently. The wing of a sparrow has the same phylogenetic origin as that of the parrot. Such a trait is homologous. This would imply that our mathematical abilities and those of other animals are the result of an adaptation in a common ancestor. Maybe the ability to count is an ancient feature of vertebrate animals, which arose in an ancestor to birds and mammals, and gave such an advantage that the trait still persists in all descendants (see e.g. Dehaene, 2000, 996). On the other hand, analogous traits are traits that are similar in function, but that do not share the same phylogenetic origin. The wing of the bat and the wing of the sparrow have evolved independently, despite their apparent similarity in function. Mathematical skills, according to this scenario, arose several times independently. This seems to be the more likely scenario, since similar mathematical skills have been found in a wide variety of species from widely diverging families (e.g. fish, reptiles, birds, and mammals). Mathematical abilities like numerosity, algorithmic behaviour and geometry are common in the mental toolkits of various animals. Number, for instance, is a natural attribute of the environment that can be discriminated by non-human animals (Davis and Pérusse, 1988, 561).

Natural selection can only operate on short term advantages, not on possible benefits in the long run. Yet, because of its accumulative effect, it can generate complex design. The classic example of the evolution of the eye is a good illustration. Clearly, an eye cannot arise at once. It can evolve in small steps, each one being slightly better than the previous, across thousands of generations, e.g. first a light-sensitive membrane (which is better than being totally blind), then gradual adjustment of the membrane to project shapes into the brain, etc. Because the eye offers its bearers an adaptive benefit, it arose several times independently (Dawkins, 1986 [1991], 77–92). A logical result is that the evolution of a brain is also a gradual process, in which each change is slight, and offers a slight adaptive benefit of its bearer. To produce a scenario for mental evolution, we must think of a small organism, very primitive, without any mental abilities. One of its descendants develops a primitive eye. To interpret the stimulus, together with the eye, a perceptual module has to arise to deal with visual stimuli. Over time, this organism develops several perceptual modules (e.g. hearing, scent) together with organs for sensory input (ears, nose). Imagine a mutation in which the organism has a shortcut in its brain with the following simple algorithm: 'if you see an animal larger than yourself, run away'. This shortcut depends for its information on input from the perceptual modules. It offers its bearer a huge adaptive benefit, and will spread quickly. A conceptual module has arisen. Over time, an animal can develop several conceptual modules to help it cope with cognitive problems in its environment (Sperber, 1996). Modules will arise in the gene pool of a population as a response on statistically recurring problems. Similarities in design can arise when the set of possible solutions to recurrent problems is limited (Hauser, 2001, 18). It is therefore not surprising that some modules are universal, since all mobile organisms have to deal with problems of a similar nature: the laws of physics, counting and navigation (algorithmic behaviour) are three sets of cognitive skills that have been selected for in many species. The *number module* discussed previously provides animals with a quick mechanism to distinguish quantities in their environment, and offers them a great advantage in making sense of the world. It enables the mind to reduce complicated forms of input (objects in time and space) to simple numerical relationships (Newberg et al., 2001, 50–51; 188–189). Both the ability to make rough estimations and exact counting of small collections of objects could help an animal to make decisions more quickly and accurately (e.g. yonder patch seems to contain more food items than the current one, so I will move to that one). Other examples include keeping track of predators or prey, the care for eggs or young, and even mate selection, e.g. in polygamous bird species, females seem to count how many females each male can attract, to choose the one which gets the most attention (Ridley, 1993 [2000]).

4.2. *The Evolution of Uniquely Human Mathematical Abilities: The Evolving Mind*

From an evolutionary perspective, every trait shared by living people (e.g. the human brain and its properties) must have arisen at some point in our evolutionary history (Tooby and DeVore, 1987). E.g. if we all walk on our hind legs, this must have arisen at some point in one of the extinct species that were ancestral to modern humans. In the case of bipedalism, the fossil record can prove this. Behavioural traits are much more difficult to attest in the fossil record: we cannot observe them directly; instead, we must infer them from the archaeological and fossil record. Uniquely human traits must have evolved in the course of hominid evolution, as a response to recurrent problems our ancestors faced (Cosmides and Tooby, 1994). To understand when and why these arose, we have to study extinct hominids and the environments in which they evolved. To examine which mathematical skills they had, we must turn to cognitive archaeology. Cognitive archaeology infers cognitive abilities and thought processes or belief systems from the past by looking at the archaeological record. The validity of the results depends of the reliability of the archaeological record, while rigorous testing – like in cognitive ethology – is not possible (Lake, 1998). If we understand which problems recurred in the course of human evolution, we can get a better understanding of how our mind is designed, and why we evolved the specific mental mechanisms it possesses today. The design or functional organization of the mechanisms present in our current cognitive architecture reflects the workings of natural selection in the past (Barkow et al., 1992; Cosmides and Tooby, 1994).

4.3. *What Mathematical Skills are Uniquely Human?*

Even though we share many mathematical abilities with other animals, humans exhibit mathematical skills that are more complex than those found in other species. Humans can count like other animals, but they can count greater quantities, and they can perform arithmetic much more accurately than other animals. Humans can do geometry, like some birds, but they can perform sophisticated geometric skills, like mental rotation and mental imaging of geometric shapes. Only humans have come up with mathematical theory. It could be argued that mathematical theory is a product of large-scale societies (China, India, Pharaonic Egypt), which have developed very recently in human evolutionary history. However, small-scale societies, hunter-gatherers, and small farming or pastoralist societies, also have mathematical theory (Ascher, 1991 [1998]). Anthropologists have documented kinship-systems in Australia and Melanesia that resemble western formal mathematical theories, with their use of axioms, propositions and a strict set of rules to make valid propositions (Ascher, 1991 [1998], 69–81).

4.4. *Complex Geometric Skills*

Human complex geometric skills are best explained as a set of mental adaptations to the unique niche *Homo* occupied since the late Pliocene: that of stone tool technology and scavenging. In the late Pliocene, a global cooling and drying event occurred around 2.7–2.6 myrs ago (deMenocal, 1995). At the same time, in East Africa, the formation of the Rift Valley blocked Atlantic moist winds to East Africa (Pickford, 1990). Because of this decline in precipitation, East Africa's dense tropical forest was gradually replaced by a more xeric environment with patchier vegetation. The Australopithecines, ancestors to all later hominid lineages, relied on high-energy plant foods, like fruits. When these were no longer abundant, selective pressure favoured new feeding strategies in hominids, like meat-eating (early *Homo*) or the consumption of large quantities of low-energy plant foods (*Paranthropus*). The genus *Homo* was the first hominid lineage to exploit animal food systematically. The oldest stone tools, found in Gona, Ethiopia, dated at 2.6–2.5 myrs ago can be linked to this change in diet (Semaw et al., 1997). Experimental archaeology shows that these tools can be made by striking one stone at a certain angle to a platform of another stone. This produces flakes (which have sharp edges) and cores. To knap stones successfully, one must position the flaking stone at a correct angle to the striking platform. If knapped successfully, a shell-like (conchoidal) flake with razor sharp edges will be flaked off. Even the oldest stone tools reveal that *Homo* had a sophisticated knowledge of fracture mechanics and deliberately turned the core while knapping (Roche et al., 1999). Micro-wear analysis on the stones and on animal bones reveal that these tools were used by scavenging hominids to obtain meat and within-bone tissue (marrow). Sharp flakes were used to scrape off meat from bones that were left by the predator that killed the animal. Hammer stones (what is left of the core when flakes have been removed) were used to break open long bones containing nutritious marrow, and to break open the skull to obtain brain tissue (Blumenschine, 1995; Capaldo, 1997). Stone tool-making abilities shaped our mathematical abilities fundamentally, because they exerted selective pressure on modules for assessing two- and three-dimensional shapes. Evidence for this comes from an experiment in which a skilled contemporary stone knapper, Nicholas Toth, had to flake simple stone tools, referred to as Oldowan technology, while his brain was being PET-scanned. The right upper parietal lobe showed a marked increase in activity while Toth was knapping. This area of the brain is commonly used to make a coherent model of external objects in space (the proper domain of this module). It uses information from perceptual modules for interpreting visual and tactile stimuli. Patients with brain damage in the right upper parietal lobe can no longer make a coherent image of the space that surrounds them (Stout et al., 2000, 1220). Although the mind not the modern knapper is not identical to that of hominids of 2.5 myrs ago, it seems reasonable to suggest that stone tool-knapping exerted selective pressures on conceptual modules that we still use in geometry nowadays (geometry has become a part of the actual domain of this module). For instance, knapping to produce a scar on the opposite face requires both actual experience of apparent motion and the ability to simulate this movement in the mind. Non-human primates have a variable and limited capacity in this field but none of them can actually perform mental rotation (Brown, 1993, 236, 241). Though some primates have exhibited variable and simple tool-use in the wild, e.g. chimpanzees use hammer stones to crack nuts (Boesch and Boesch, 1993), they have no specialized modules to deal with tool-use. They do not understand the dynamics of stone tool knapping, as is shown in an experiment in which the bonobo Kanzi was encouraged to make stone tools to cut off a rope around a box containing a food reward. Kanzi repeatedly failed to produce sharp flakes, because he did not understand the knapping process (Toth et al., 1993).

Hand axes, stone tools that appear in the archaeological record at about 1.4 myrs ago, and dwindle away at about 250, 000 years ago, show an even better understanding of geometry and fracture mechanics compared to the earlier stone tools because of their constancy in design: they are always shaped like a teardrop, and are bilaterally symmetrical. Hand axes exert even more from the geometric imaging device in the upper right parietal lobe, since its shape is fixed. Some archaeologists (e.g. Noble and Davidson, 1993; McPherron, 2000) have argued that the constant shape of the hand axe is the result of the knapping process, and that the hominids who made it did not intend to make a symmetrical shape. The evolution of the hand axe makes this unlikely, since its shape becomes progressively symmetrical and ever more precise. After 500, 000 BP the trimming (accurate removal of small flakes to produce symmetry) is very extensive, and some of the removed flakes are not useful as tools. Also, the bilateral symmetry extended in three dimensions: it extends to all the cross-sections of the artefacts, including cross-sections oblique to the major axes (Wynn, 2000).

4.5. *Complex and Accurate Numerical Competence*

To understand why humans are very good at dealing with numbers and calculations, we have to examine numerical skills cross-culturally. There are languages with very few words for numbers; they have for instance only numerical words for one, two, many. Most of these cultures do not trade extensively, so an easy to handle vocabulary for numbers is not necessary. In every culture, people use external means to represent numerosities. In some cultures, e.g. the Aranda (Australian aborigines), lines drawn in the sand, sticks or pebbles are used to indicate numerosities. In other cultures, parts of the body (sometimes just the fingers and toes, sometimes all parts of the body) correspond to numerosities (Ifrah, 1985). Thus, an indigenous Papua New Guinean may not have a specialized word for 16, but he can say 'right little toe' to represent 16 (Butterworth, 1999a, 54–58). What is important for counting is that a fixed order of words or items is used in a one-to-one correspondence with the items to be counted (ordinality) Thus, we cannot count a collection of, say 5 objects, if we do not keep the order of Arabic numerals fixed (e.g. 2, 10, 8, 6, 4 => the result of the counting (4) is wrong). If the words are parts of the body, used in a fixed order, they can be used to count as well as Arabic numerals. This externalization (body parts, tallies, or pebbles) is the key to explain why humans are so good at counting and performing other cognitive tasks. Humans can externalize processes of their brain into the outside world, e.g. on their bodies. Externalization expands the capacity of the individual human brain, because it can be used to remember, calculate and reason better than an individual brain can. The first solid evidence for the externalization of thought-processes occurs at about 40, 000 BP in the archaeological record. These are objects in bone or antler, incised with regular patterns of notches. Undoubtedly, they have been used to count, or to keep track of cyclical patterns in the environment. Marshack (1972) thinks they represent lunar calendars. d'Errico (1995) devised the more general term *Artificial Memory System*. The notched objects in bone or antler are devices to remember numerical patterns, e.g. counting of group size, or timing of animal migratory events. It is no coincidence that evidence for more efficient fishing and hunting occurred at the same time as the first *Artificial Memory Systems* appeared in the archaeological record in Africa and Europe, at about 40, 000 BP. Coastal sites of South Africa (Blombos Cave, Klasies River Mouth and Die Kelders Cave 1) show an abrupt change in hunting and fishing techniques at this time (the Middle Stone Age to late Stone Age transition, being the ascent of the culturally modern *Homo sapiens*). From the ages of the fur seals they ate, we can infer that LSA people timed their coastal visits to the August-to-October interval, when nine to ten month-old seals could be easily captured on the shore and when resources inland were at their poorest (Klein and Edgar, 2002, 239). This implies an understanding of recurrent patterns (cyclicity) which is not found in earlier hominids, and which could only be facilitated with *Artificial Memory Systems*. The evolutionary advantage of AMS is that humans were able to exploit their environment more efficiently, as soon as they could externalize and store cyclical patterns outside their brains.

4.6. *Mathematical Theory*

Some mathematicians might argue: this Darwinian approach might tell us something about some basic human and animal mathematical abilities, but it can never explain the scope and power of mathematical theory. How could we explain, for example, counter-intuitive concepts, like infinity or $\sqrt{2}$? How to explain the fit and surprising user-friendliness of mathematical theories, which can be frequently used for purposes for which they were originally not intended? This feeling of awe is the main reason why so many mathematicians are Platonists: they feel as if mathematical ideas lay dormant, waiting to be discovered. Such feelings are not unique to mathematics: musicians, painters and writers also experience that their work is an existence in its own right and that, in some ways, it transcends them. Instead of assuming Platonic worlds for painting, mathematics, or music, it might be wiser to ask whether creative thought processes give rise to these feelings. Creativity is essentially a neurological process in which humans connect old ideas to form new ideas, or in which they transform an existing set of ideas (a conceptual framework) so radically, that a completely new set of ideas can arise. Creative people, mainly scientists and artists regularly experience that this arises naturally and unconsciously, in such unlikely places as their bed, bath or bus (Boden, 1990). On a neurological level, creativity might be the result of random firing of neurons, resulting in the connection of two or more sets of previously stored domain-specific knowledge areas, thus giving rise to a new idea. This random firing also occurs in other animals, where it generates unpredictable behaviour. Sometimes this unpredictability offers a huge evolutionary advantage, e.g. a rabbit makes random jumps if pursued by a predator, and can escape because its course cannot be predicted (Miller, 2000, 392). Hominids, in the course of their evolutionary history, have heavily relied on their cognitive capacities.

How can mathematical theory be reconciled with modular thinking? If every thought we generate can be described as the output of a specialized cognitive circuit, how can we explain mathematical theory? Some human behaviours, like playing tennis, chess, science, religion, and mathematical theories have arisen too recently in human cultures to be the direct product of natural selection. As we have seen, natural selection can only work slowly over many generations in response to statistically recurrent problems. Of course, one could always concoct a story in which a novel-looking trait (such as music) seems a biological adaptation. A typical just-so story would run like this: music was favoured by natural selection, because mothers who could sing their babies to sleep attracted less predators. However, such application of the evolutionary approach is ill founded, because it merely assumes the adaptive character of a trait without a plausible demonstration. To resolve this problem, I turn to Sperber's distinction between *proper* domains, actual domains and cultural domains for modules. Modules respond to each kind of stimulus to which they have evolved to respond to, e.g. numerical tasks activate a number module. Since modules do not know whether this stimulus is actually one for which they are evolved, they will respond to any stimulus that resembles their proper domain. It can be inferred from Martin et al. (1996) that PET-scans of subjects looking at drawings of animals are not unlike those of subjects looking at real animals. The set of all possible stimuli to which a module will respond constitutes the *actual domain* of the module. Most animals depend on information they acquired by themselves. In test-situations where animals were forced to choose between acquiring information from conspecifics or by themselves, they always choose to acquire information by themselves (Giraldeau et al., 2002). Humans, on the other hand, rely very heavily on information they get from others. (Consider, for instance, that almost everything you know about this planet stems from information provided by others and not from first-hand experience). This is partly because we have a complex communication system, which enables us to express an unlimited variety of thoughts, and partly because we have ways to store information externally (in the form of AMS). Thus, a module can be triggered by information that is provided by other people, this is its *cultural domain*. In memetics, bits of cultural information are replicators (like genes) who compete for space inside our heads, and which (like genes) have differential reproductive success (see e.g. Dawkins, 1989; Blackmore, 1999). Memetics can explain why some ideas are very widespread, while others remain virtually unknown. E.g. horoscopes are extremely successful: they are well known and practiced enthusiastically in great parts of Europe, America, India and China, and many people firmly believe that celestial bodies influence their lives. Conversely, mathematical theory is poorly transmitted. Many children find mathematics difficult and unrewarding, and are discouraged to practice mathematics later in life. Why is mathematical theory so unsuccessful in transmission (in memetical terms: why is it unfit?). Sperber (1996) proposes that the success of any cultural trait depends on its fit with the innate information of a module. As we have seen, modules are not empty but provide additional information to specific kinds of stimuli. If a cultural trait matches this innate information very well, it has a greater chance of success. For instance, in Western culture, laypeople can easily acquire information about dinosaurs. Even though we will never see a living dinosaur, we can easily grasp inferred information about these animals, because they match our ideas about animals, e.g. dinosaurs are reptiles (body-plan), lay eggs (mode of reproduction), and can be carnivorous or herbivorous (diet). On the other hand, some information provides a severe violation to innate modular information. Because ideas that violate this information are extremely arresting, they also have a great tendency to be transmitted. Boyer (2001) has argued that religious ideas are so widespread, *because* they violate some types of innate modular information, while being in strong concordance with others. Ghosts, for instance, are people, and they behave as other people do (they have motives, desires, they can get angry or be pleased). This is in accordance with modules we have developed to interact with other people, called *Theory of Mind Modules*. However, ghosts also violate innate physics, because they can go through walls, and appear and disappear at will. This mind grabbing mixture of intuitive and counterintuitive ideas appears to be so irresistible, that concepts about ghosts will readily spread in any cultural group. This explains why religious beliefs are universal, and why novel religious ideas can become successful in any society, e.g. reincarnation has become popular in current Western folk beliefs. Other cultural ideas provide a less easy fit or a less obvious violation to a specific module. The module will not respond very well, or will only do so after years of intense tutoring and training. Mathematical theory is such a cultural trait. Though we have innate mechanisms to deal with number, geometry, and algorithms, it is hard to combine these abilities in a formal framework. Apparently, we do not usually need to combine and formalize our mathematical skills, and evolution has not provided us with any shortcuts to do so. It does not follow that mathematical theory is non-modular (since all thinking is modular according to the *Mass Modularity* hypothesis) – it does mean that humans are not genetically disposed to deal with mathematical theory, as they are with interpersonal interaction, counting or tool use. The fact that a trait is not the result of evolved properties does not make it less valuable or less universally applicable. The scientific method is also a very rare and recent cultural trait that has only arisen once, in Western culture, in the course of the seventeenth century. No one would deny its universality and applicability. In the same way, mathematical theories are very useful as models to describe patterns in this world.

A final question might be why mathematical theory is so apt to describe the world. Mathematical theories have successfully been applied in physics, biology, economics and psychology. This is because the world is not chaotic, but has recurring patterns. Like other animals, humans can recognize these patterns. Mathematical theory is a tool for describing patterns accurately – it is about relationships between abstract entities. It follows that these abstract descriptions of patterns can be applied to patterned phenomena in the world. Human mathematical theory is much more complex and far ranging than animal mathematical abilities, because human brains are much more specialized and complex. Furthermore, humans can accumulate knowledge in AMS (e.g. books), for instance about actual patterns in the world, or models abstracting these patterns (e.g. mathematical theory). The creativity humans show in making connections between these abstract models, and the patterns they perceive in reality, is caused by the (sometimes random) neural shooting between domain-specific sources of information.

5. CONCLUSIONS

I have reviewed extensive empirical research indicating that several mathematical abilities are present in infants and other animals. There is abundant convergent evidence from cognitive ethology, developmental psychology and neuropsychology that these abilities are the result of category-specific neural circuits (modules). I have also shown that a Darwinian (adaptationist) approach is theoretically the most plausible way to explain these modules. The evolution of human mathematical abilities is still little understood. In this paper, I have suggested some possible ways in which conceptual frameworks from evolutionary biology, cognitive archaeology and cultural evolution studies can contribute to a reconstruction of the evolution of human mathematical abilities.

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